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Interaction of anti-plane wave with an edge crack in a transversely isotropic strip

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Abstract

In this monograph, a model was proposed to analyze anti-plane wave propagation through a transversely isotropic strip featuring an edge crack. Dual integral equations were formulated to solve the governing mixed boundary value problem using the Hankel transform technique. These dual integral equations were then converted into a second-kind Fredholm integral equation through Abel's transformation. Numerical calculations for the stress intensity factor (SIF) were carried out with graphical demonstration. To establish the impact of the medium inspected and normalized strip width on the SIF, two transversely isotropic materials, namely Graphite-Epoxy and Hafnium were used.

Keywords: Transversely isotropic strip; Anti-plane wave; Finite edge crack; Dual integral equation; Stress intensity factor.

1. Introduction

A material with one axis of symmetry in addition to three planes of symmetry is referred to as transversely isotropic (hexagonal) [1]. To describe a transversely isotropic media, it is needed to specify five material constants mainly C_{11}, ,C_{12},C_{13},C_{33},C_{44}. Due to the stiffness and strength of these materials, investigating elastic wave propagation in composite materials with cracks is highly important for the non-destructive characterization of the damage state in composite like transversely isotropic materials such as Graphite-Epoxy, E glasss Epoxy, Hafnium etc [2]. This investigation helps to measure the initiation and growth of cracks under antiplane wave propagation.

The scattering of a harmonic longitudinal wave by a penny-shaped crack in a transversely isotropic material has been investigated using Hankel transform techniques by Tsai [2]. The interaction of plane time-harmonic SH-waves with microcracks in transversely isotropic materials have been presented using Fourier integrals, Chebyshev polynomials, and the Galerkin method by Zhang and Gross [3]. Noriss and Achenbag [4] analyzed diffraction of elastic wave by a Semi-Infinite crack in a transversely isotropic material adopting fourier Integral method and Weiner-Hopf method. Ting [5] has found surface

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wave solution for anti-Plane shear surface waves in anisotropic half-space with depth-dependent material properties. Brock [6] has considered rolling without slip by a rigid cylinder on a transversely isotropic, coupled thermoelastic half-space at constant subcritical speed.

Evidently, an analysis of stress behaviour in transversely isotropic strips having an edge crack subjected to antiplane wave propagation has not outlined earlier. In this monograph, a model was proposed to analyze anti-plane wave propagation through a transversley isotropic strip featuring an edge crack. Dual integral equations were formulated to solve the governing mixed boundary value problem using the Hankel transform technique. These dual integral equations were then converted into a second-kind Fredholm integral equation through Abel's transformation. Numerical calculations for the stress intensity factor (SIF) were carried out with graphical demonstration. To embelish the impact of the medium inspected and normalized strip width on the SIF, two transversley isotropic materials, namely Graphite-Epoxy and Hafnium were used.

2. Formulation and Solution of the Problem

Consider an infinitely long transversley isotropic strip with finite width I with an edge crack of length C under anti-plane wave propagation. An edge crack of finite length is located (Figure 1) at $0 \leq X \leq C,\ -\infty < Z < \infty,\ Y = 0$ referring to axes $(X,\ Y,\ Z)$ of the Cartesian coordinate. The crack's location becomes $0 \leq x \leq 1,\ -\infty < z < \infty,\ y = 0$ by normalizing all lengths with respect to b by letting $\frac{X}{C} = x, \frac{Y}{C} = y, \frac{I}{C} = H$. The plane of inspected model is taken to be the $x - y$ plane. Let the edge crack is influenced due to time-harmonic anti-plane incident normally along direction of the positive $y-$axis.

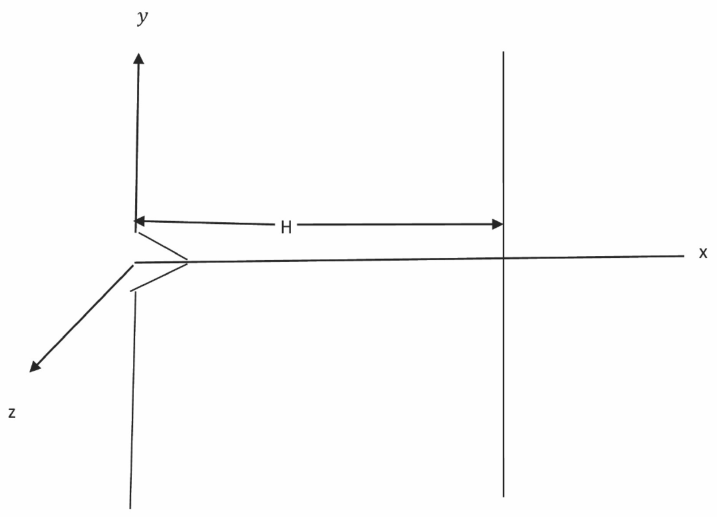

Figure 1: Geometry of transversely isotropic strip

Due to the anti-plane wave motion, the displacement components can be considered as $u = 0,\ \ v = 0,\ \ w = w(x, y, t)$. For an transversley isotropic medium, the anti-plane equation of motion takes a form similar to that below:

$$\frac{C_{11} - C_{12}}{2}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} = \frac{C^2}{c_s^2}\frac{\partial^2 w}{\partial t^2}. \tag{1}$$

where C_{11}, C_{12}, C_{44} are elastic constant of the materials taken, and $c_s = \sqrt{\frac{C_{11}-C_{12}}{2\rho}}$ is SH-wave velocity depending on the transversely isotropic medium.

By substituting $w(x, y, t) = w(x, y)e^{-i\omega t}$, the Equation (1) is converted to

$$C_{66}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} + \frac{C^2\omega^2}{c_s^2}w = 0. \tag{2}$$

where $C_{66} = \frac{C_{11}-C_{12}}{2}$. The nonzero component of the stress tensor can be expressed as follows:

$$\begin{aligned}\sigma_{xz}(x, y) &= C_{66}\frac{\partial w(x, y)}{\partial x},\\ \sigma_{yz}(x, y) &= C_{44}\frac{\partial w(x, y)}{\partial y}.\end{aligned}$$

Application of the standard Hankel integral transform, Equation (2) admits solution in the form

$$w(x, y) = \int_{-\infty}^{\infty} E(\gamma)e^{-\chi y}e^{i\gamma x}d\gamma + \int_0^{\infty}\Big[F(\eta)e^{\beta x} + G(\eta)e^{-\beta x})sin(\eta y)\, d\eta, \tag{3}$$

with

$$\begin{aligned}\chi &= \sqrt{M(\gamma^2 - k_s^2)}, \quad \gamma > k_s,\\ &= -i\sqrt{M(k_s^2 - \gamma^2)}, \quad \gamma < k_s,\end{aligned}$$

and

$$\begin{aligned}\beta &= \sqrt{\frac{1}{M}(\eta^2 - k_s^2)}, \quad \eta > k_s,\\ &= -i\sqrt{\frac{1}{M}(k_s^2 - \eta^2)}, \quad \eta > k_s,\end{aligned}$$

and $M = \frac{C_{66}}{C_{44}},\ k_s^2 = \frac{C^2\omega^2}{c_s^2 C_{66}}$.
The boundary conditions at $y = 0$ are

$$\sigma_{yz}(x,0) = -\sigma_0, \quad 0 \leq x \leq 1, \tag{4}$$

$$w(x,0) = 0, \quad 1 \leq x \leq H. \tag{5}$$

The boundary conditions on the edges $x = 0$ and $x = H$ are

$$\sigma_{xz}(0,y) = 0, \quad |y| < \infty, \tag{6}$$

$$\sigma_{xz}(H,y) = 0, \quad |y| < \infty. \tag{7}$$

The expressions for stresses $\tau_{\theta z}$ now becomes

$$\sigma_{yz}(x,y) = -C_{44}\int_{-\infty}^{\infty} \chi E(\gamma)e^{-\chi y}e^{i\gamma x}d\gamma + C_{44}\int_0^{\infty} \eta\Big[F(\eta)e^{\beta x} + G(\eta)e^{-\beta x}\Big]cos(\eta y)d\eta, \tag{8}$$

$$\sigma_{xz}(x,y) = iC_{44}\int_{-\infty}^{\infty} \gamma E(\gamma)e^{-\chi y}e^{i\gamma x}d\gamma + C_{44}\int_0^{\infty} \beta\Big[F(\eta)e^{\beta x} - G(\eta)e^{-\beta x}\Big]sin(\eta y)d\eta. \tag{9}$$

Employing Abel's transformation, we obtained the second kind Fredholm integral equation as following

$$h(\alpha) + \int_0^1 \nu h(\nu)N(\mu,\alpha)d\nu = 1, \tag{10}$$

where the kernel $N(\nu,\alpha) = N_1(\nu,\alpha) - N_2(\nu,\alpha) - N_3(\nu,\alpha)$.
Applying the process of contour integration technique, the integral $M_1(\nu,\alpha)$ can be transformed into integral with finite limit as

$$N_1(\nu,\alpha) = -ik_s^2\int_0^1 \sqrt{1-\epsilon^2}J_0(k_s\epsilon\nu)H_0^{(1)}(k_s\epsilon\alpha)d\epsilon, \tag{11}$$

$$N_2(\nu,\alpha) + N_3(\nu,\alpha) = -\int_0^{k_s} \frac{\lambda^2 J_0(\beta'\nu)J_0(\beta'\alpha)e^{i\beta' H}}{\beta' sin(\beta' H)}d\lambda + \int_{k_s}^{\infty} \frac{\lambda^2 I_0(\beta\nu)I_0(\beta\alpha)e^{-\beta H}}{\beta sinh(\beta H)}d\lambda, \tag{12}$$

where $\beta' = \sqrt{\frac{1}{M}(k_s^2 - \lambda^2)}$.

3. Stress Intensity Factor

Anti-Plane stress $\sigma_{yz}(x,0)$ around the crack can be described as

$$\sigma_{yz}(x,0) = \frac{\sigma_0 x}{\sqrt{x^2-1}}h(1) + O(1), \qquad |x| > 1. \tag{13}$$

The stress intensity factor (SIF) is a physical quantity that describes the stress state in a structure with a crack. SIF for anti-plane wave is

$$K_{III} = \lim_{x\to 1^+} \Big|\frac{\sqrt{x-1}\sigma_{yz}(x,0)}{\sigma_0}\Big|. \tag{14}$$

Finally, using Equation (10), the expression for SIF become

$$K_{III} = \frac{|h(1)|}{\sqrt{2}}. \tag{15}$$

4. Discussion of Numerical Outcomes

To illustrate the outcomes graphically, we calculated the numerical values of the stress intensity factor (SIF) using Equation (15). Numerical computations were performed using the Fox and Goodwin[7] and the Gauss quadrature rule, implemented with MATLAB software. I considered the numerical values of elastic constants for two sets of transversely isotropic medium, as listed in Table 1.([3, 8]).

Table 1: Elastic constants of transversely isotropic medium

Elastic medium	$C_{44}(GPA\ unit)$	$C_{66}(GPA\ unit)$
Graphite-Epoxy	3.39	6.16
Hafnium	5.57	5.2

Since the stress values near a crack tip are always very high (and approach infinity at the tip), the strength-of-materials approach to failure prediction the material fails when the stress exceeds some critical value. In the fracture mechanics approach, material failure is predicted by comparing the stress intensity factors with a critical value, K_c, rather than comparing the maximum stress value with a critical stress value. This critical value is known as the critical stress intensity factor or the fracture toughness of the material [1].

Here, numerical values of the normalized stress intensity factor (SIF) $((K_{III}))$ against the dimensionless frequency (k_s) are depicted through graphs in Figs. 2-3. To examine the domination of normalized strip width $(H = \frac{I}{C})$ and the transversely isotropic medium on the SIF, I considered three different values of strip width: $(H = 2.5), (H = 3.5)$, and $(H = 4.5)$.

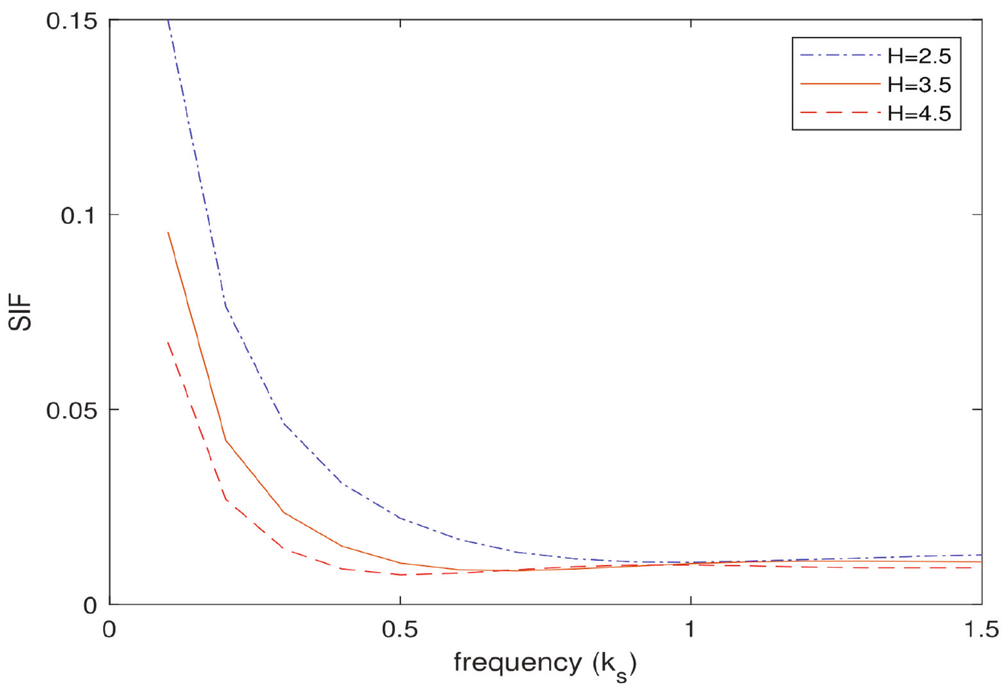

Figure 2: Plot of SIF vs. frequency k_s for Graphite-Epoxy

For all values of the dimensionless frequency (k_s), it appears that as the strip width (H) decreases, the peak value of the SIF gradually increases. Additionally, based on Figs. 2-3, we can conclude that the SIF curve reaches a peak value first and then gradually decreases as the frequency increases.

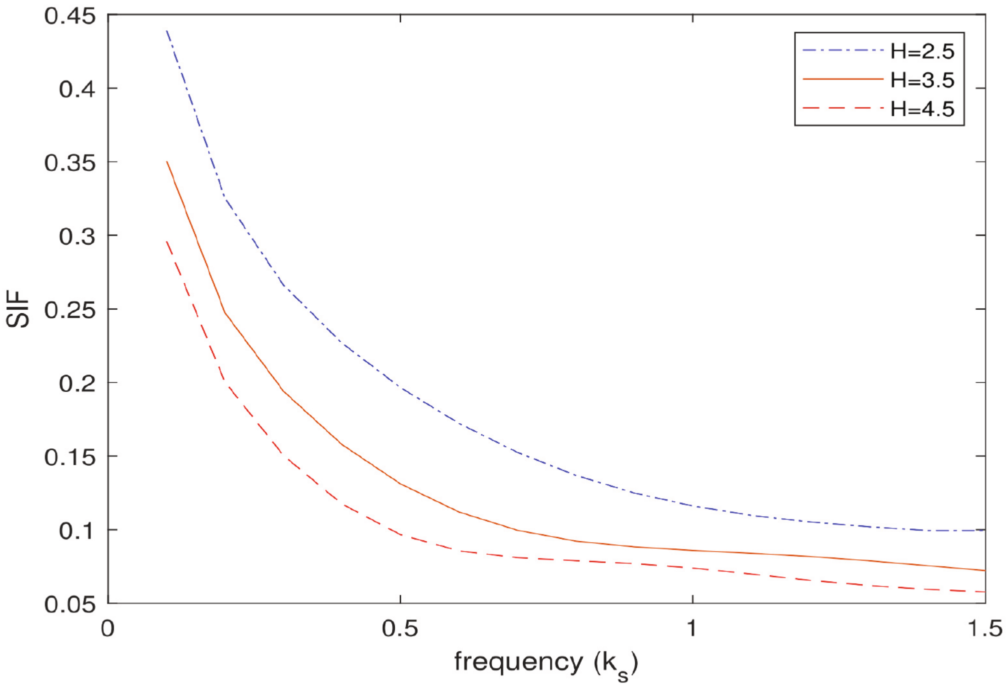

Figure 3: Plot of SIF vs. frequency k_s for Halfnium

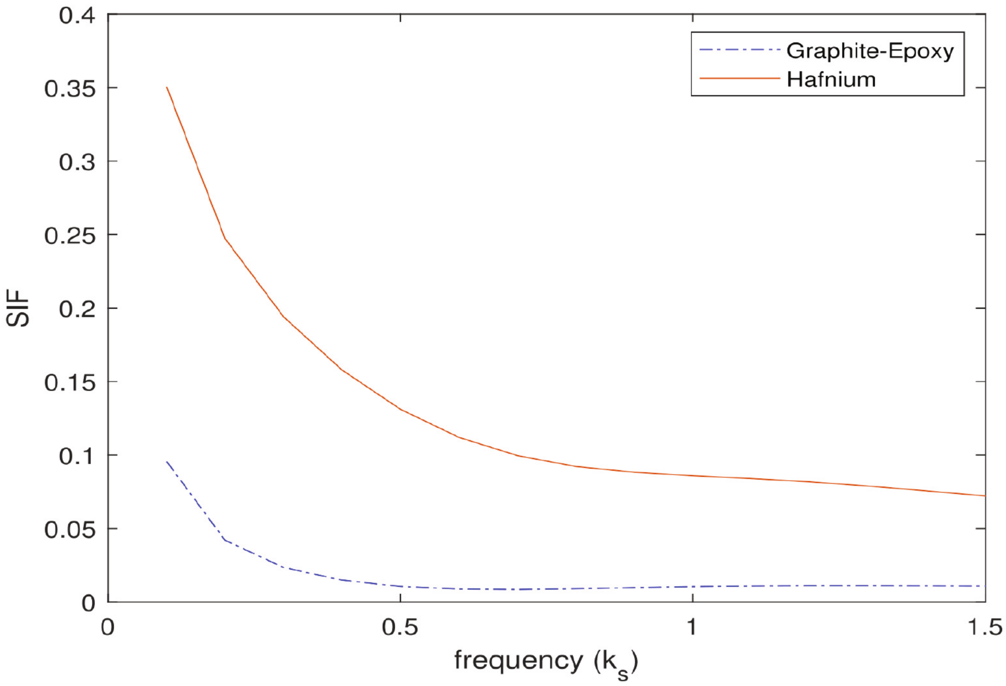

Figure 4: Plot of SIF vs. frequency k_s with strip width H = 3.5

In Fig. 4, SIF values are portrayed for two distinct transversely isotropic media (Hafnium and Graphite-Epoxy) as a function of frequency (k_s) with a fixed strip width $(H = 3.5)$ to observe the effect of the transversely isotropic nature on SIF. It can be observed that the peak value of the SIF at the crack tip for Hafnium (Fig. 4) is higher than that for Graphite-Epoxy at the same frequency (k_s). Therefore, we can conclude that the Graphite-Epoxy medium appears to be elastically stronger compared to the Hafnium medium. Therefore, the peak value of the SIF can be adjusted by varying the strip width and the type of transversely isotropic medium.

5. Conclusion

Generally, it is observed that crack begins to propagate within engineering solids when the SIF values exceed a certain limit (depending on nature of medium) termed critical SIF. In structural engineering, the main aim is to resist the onset of crack within a composite medium to avoid damage to solid bodies by controlling SIF within a specific range that is critical SIF. Considering this fact, we may conclude that SIF values might be controlled by monitoring the width of the strip and geometric parameters of the transversely isotropic medium, which is rightly envisioned in fracture analysis. In a practical sense, this observation may help to select material to make a solid with most elastic resistance to fracture [9]. This study can play important role in controlling failure mechanisms, failure propagation, fracture toughness, and the overall stress-strain behavior .

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